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Analysis of regenerative enthalpy exchangers

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Abstract—A computationally simple model of a solid desiccant air-to-air enthalpy exchanger is developed. The theory of equilibrium exchange systems is used to establish the operating conditions under which enthalpy exchange between the two flow streams can be accomplished. To achieve maximum enthalpy exchange between the two air streams, the regenerator must be operated at conditions such that neither of the two transfer waves reaches the outlet of the enthalpy exchanger. Comparison with the numerical solution of the coupled equations for finite transfer coefficients shows that the product of two non-dimensional parameters, $\bar{\gamma}_1\Gamma$, must be greater than 1.5 in order to operate the enthalpy exchanger at a point where the enthalpy exchange effectiveness is determined only by the number of transfer units. The case of infinite rotation speed is used as a basis for developing a computationally simple measure of performance. The outlet states are computed using the ε–*NTU* correlations for counterflow direct transfer heat exchangers. It is found that these correlations are accurate at operating conditions where enthalpy exchange occurs. The predicted outlet states lie, in the case of unity Lewis number, on a straight line connecting the outlet states.

1. INTRODUCTION

IN REGENERATIVE heat exchangers, also called periodic flow heat exchangers [1], heat is transferred from the hot fluid during the first period to a solid energy carrier and, during the second period, from the solid to the cold stream. The governing heat transfer mechanism is convection. Two different combinations of regenerator type and flow arrangement are used to accomplish the alternating heating and cooling of the matrix. The valved type alternates the two flow streams through a fixed bed. Continuous operation is possible using two regenerators. The rotary type permits continuous operation using only one matrix by rotating the matrix cyclically from one stream to the other.

Regenerative heat exchangers are also mass exchangers if phase changes of one or more components of the fluid streams occur within the thermodynamic operating conditions. There are two possible mass transfer mechanisms that can occur in heat exchange operations: condensation/evaporation and sorption/desorption. Both mechanisms are described with the same differential equations, but with different thermodynamic relationships. Condensation and evaporation can occur using any type of matrix. The sorption processes require a matrix carrying sorptive particles such as silica gel or lithium chloride crystals. For this type of matrix, the components are transferred in an adsorbed rather than a condensed phase and the regenerator therefore shows a different behavior.

Rotary regenerators carrying hygroscopic materials are used in air dehumidification and energy recovery. When a regenerative heat and mass exchanger is used as a dehumidifier, the desired outlet state of the process stream is a state of low absolute humidity. The change in enthalpy of the two streams is very small for well designed dehumidifiers. In energy recovery applications, however, the goal is to have the outlet state of one stream as close as possible to the inlet state of the other stream. Therefore, a high enthalpy effectiveness is desired and the regenerator is referred to as an enthalpy exchanger.

Several models have been developed for adsorptive heat and mass exchangers [2–6] as well as for non-hygroscopic devices with mass transfer in the condensed phase [7–10]. Most of the previous studies on adsorptive regenerators have not considered the conditions relevant for enthalpy exchangers, focusing instead on the analysis of the various solid desiccant cycles using rotary sensible heat exchangers as well as regenerative dehumidifiers.

This study presents an analysis of enthalpy exchangers used for energy recovery in combination with conventional air conditioning systems. The fluid streams are treated as binary mixtures of air and water vapor. Partial differential equations are used to describe the exchanger performance. In order to calculate the outlet states for a given set of system parameters and inlet conditions, a finite difference scheme has been developed by Maclaine-cross [3] and implemented in a FORTRAN code called MOSHMX. The finite difference solution requires considerable CPU time and is not suited for long-term system simulations. A simplified method based on the effectiveness correlations for a counterflow direct type heat exchanger is presented which predicts the outlet states of a well designed enthalpy exchanger. MOSHMX was used extensively in order to establish

NOMENCLATURE

a	derivative of a thermodynamic state property function
A_j	heat and mass transfer area at period j
c_A	fluid specific heat
c_m	matrix specific heat
C^*	ratio of minimum to maximum air capacitance rate
F_1	first combined heat and mass transfer potential
F_2	second combined heat and mass transfer potential
h_t	heat transfer coefficient
h_w	mass transfer coefficient
i_f	specific enthalpy of humid air
I_m	specific enthalpy of the matrix
I_w	specific enthalpy of water vapor
i_v	specific heat of vaporization
i_s	specific differential heat of adsorption
L	flow length of the rotary regenerator
Le	Lewis number, NTU_{tj}/NTU_{wj}
$\dot{m}_f$	mass flow rate of dry air
M_m	mass of the dry matrix
NTU_{tj}	number of transfer units for sensible heat transfer (period j), $A_j h_{tj}/\dot{m}_{fj}$
NTU_{wj}	number of transfer units for mass transfer (period j), $A_j h_{wj}/\dot{m}_{fj}$
NTU	number of overall transfer units for the counterflow direct type: $NTU_{tj}/2$, for heat transfer; $NTU_{wj}/2$, for mass transfer
t_f	temperature of humid air
t_m	temperature of the matrix
T	time required for one regenerator rotation
w_f	humidity ratio of humid air
w_m	humidity ratio of humid air in equilibrium with the matrix
w_{sat}	humidity ratio of humid air at saturation
W_m	water content of the matrix
W_{max}	maximum water content of the matrix
x	flow coordinate
z	dimensionless flow coordinate, x/L.

Greek symbols

β_i	time fraction of period j, T_j/T_i
$\bar{\gamma}_i$	averaged combined capacitance ratio
Γ_i	ratio of 'matrix flow rate' to air flow rate (period j), $M_m/\dot{m}_{fj}T$
ε_i	enthalpy transfer effectiveness
ε_t	heat transfer effectiveness
ε_w	mass transfer effectiveness
Θ	time coordinate
λ_i	dimensionless wave speed of the ith transfer wave
Φ	dimensionless time coordinate, Θ/T.

Subscripts

DA	dry air
DM	dry matrix
f	fluid or air state
i	1 or 2, index for the two combined heat and mass transfer potentials
in	inlet state
j	1 or 2, index for stream or period
$3-j$	index for the 'other' stream or period
m	matrix state
out	outlet state
sat	saturated state
w	water.

the range of system parameters and inlet conditions where the computationally simple method applies. A parametric study is presented in order to examine the effect of mass flow rate, rotation speed, heat and mass transfer coefficients and the desiccant properties on the performance of enthalpy exchangers.

2. MODEL FORMULATION

Figure 1 illustrates a rotary regenerator with a matrix which is cyclically exposed to physically separated air streams in counterflow arrangement. The regenerator is rotating between two ducts which are separated by a wall to prevent mixing of the two air streams. The two streams are referred to as stream or period 1 (in this study, the stream with the lower temperature) and stream or period 2. The analysis is based on the following conventional assumptions.

(1) The state properties of the inlet streams entering the regenerator are steady and uniform in radial and angular position at the inlet face.

(2) There is only a small pressure drop along the axial flow length compared to the total pressure; the

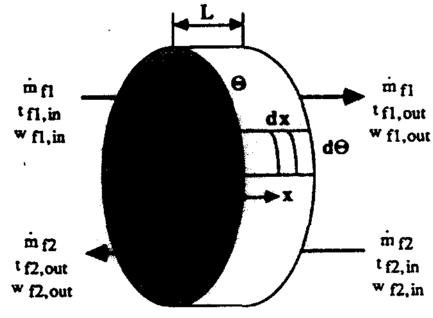

FIG. 1. Coordinate system and nomenclature of a rotary regenerator.

changes of thermodynamic properties of the fluid and matrix are not affected by this small pressure drop.

(3) The mass of air entrained in the matrix is small compared to the mass of the matrix. Therefore, the energy storing effect of the air is neglected as well as the carry-over of air when switching from one period to the other.

(4) The mixing of the fluid streams through leakage past radial seals is neglected.

(5) Angular and axial heat conduction and water diffusion due to gradients in temperature and concentration, respectively, are neglected.

(6) The matrix is considered to be a homogeneous solid with constant matrix characteristics, constant porosity and uniform properties in the radial coordinate.

(7) The heat and mass transfer between the air streams and the matrix can be described by overall convective transfer coefficients which are constant throughout the system.

(8) The regenerator operates adiabatically overall.

The conservation and transfer rate equations can be written as:

energy

$$\frac{\partial i_f}{\partial z}+\Gamma_j\beta_j\frac{\partial I_m}{\partial \Phi}=0 \tag{1}$$

$$\frac{\partial i_f}{\partial z}=c_A\,NTU_{tj}(t_m-t_f)+i_w\,NTU_{wj}(w_m-w_f); \tag{2}$$

mass

$$\frac{\partial w_f}{\partial z}+\Gamma_j\beta_j\frac{\partial W_m}{\partial \Phi}=0 \tag{3}$$

$$\frac{\partial w_f}{\partial z}=NTU_{wj}(w_m-w_f). \tag{4}$$

Equations (1)–(4) form a set of coupled hyperbolic differential equations. The coupling is a result of two different effects. First, there is both a sensible and a latent term in the energy transfer rate equation. Furthermore, the equations are coupled through the thermodynamic relationships between enthalpy, temperature, and water content:

$$i_f=i_f(t_f,w_f) \tag{5}$$

$$I_m=I_m(t_m,W_m) \tag{6}$$

$$W_m=W_m(t_m,w_m) \tag{7}$$

$$i_w=i_w(t_f). \tag{8}$$

Following ASHRAE [11], humid air at ambient pressure is treated as an ideal gas mixture of dry air and water vapor. Equation (7) is the equilibrium relation between the water content of the matrix and the humidity ratio of the air and is known as the adsorption isotherm correlation. The enthalpy of the matrix in equation (6) is determined in part by the adsorption isotherm correlation, the specific heat capacity of the matrix material and the energy effect involved in the adsorption process, referred to as heat of adsorption. Two different adsorption isotherm correlations were used in this study. The Dubinin–Polanyi theory was investigated by Van den Bulck [12] to describe the adsorption characteristics of silica gel whereas a generic isotherm model developed by Jurinak [2] was used for molecular sieves and Brunauer Type 3 adsorbents.

Equations (1)–(8) form a set of differential and algebraic equations with eight thermodynamic state properties as the dependent variables and the non-dimensional axial flow direction z and time Φ as the independent variables. The boundary conditions of the system are:

$$\text{at } z=0 \quad w_f(z=0,\Phi)=w_{fj,\text{in}} \quad j=1,2$$

$$t_f(z=0,\Phi)=t_{fj,\text{in}} \quad j=1,2$$

at $\Phi=\beta_j$

$$\lim_{\Phi\to\beta_j^-} w_m(z,\Phi)=\lim_{\Phi\to\beta_j^+} w_m(z,\Phi) \quad j=1,2$$

$$\lim_{\Phi\to\beta_j^-} t_m(z,\Phi)=\lim_{\Phi\to\beta_j^+} t_m(z,\Phi) \quad j=1,2.$$

The last two equations are called reversal conditions and imply that, at steady state operation, the matrix state at the end of one period must be equal to the matrix state at the beginning of the other period. The reversal condition is the reason for the substantial difference in the solution techniques of the fixed bed and the rotary regenerator since iterative methods are required in order to find a solution which matches the reversal condition.

The coupled non-linear system of equations (1)–(8) cannot be solved analytically and therefore numerical or approximate analytical solution techniques must be used to determine the outlet states as well as the state property distributions of the fluid and the matrix.

3. SOLUTIONS FOR UNCOUPLED SITUATIONS

The energy and mass equations may be uncoupled under certain sets of operating conditions. In the case of infinite transfer coefficients, the regenerator becomes an equilibrium exchange system and can be described solely by the conservation laws and thermodynamic relationships. Van den Bulck *et al.* [4] developed a wave model for determining the outlet stream states of ideal (infinite transfer coefficient) regenerative heat and mass exchangers. The conservation laws can be transformed into a set of uncoupled kinematic wave equations using combined heat and mass transfer potentials instead of the enthalpies and humidity ratios:

$$\frac{\lambda_1}{\Gamma_j}\frac{\partial F_1}{\partial z}+\frac{\partial F_1}{\partial(\Phi/\beta_j)}=0 \tag{9}$$

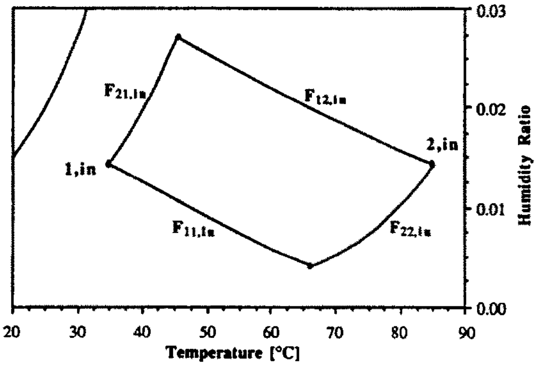

FIG. 2. Regenerator inlet states with lines of constant F-potential.

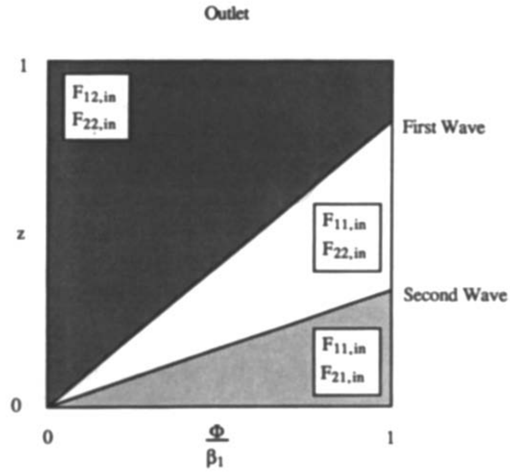

FIG. 3. Wave diagram for period 1 of an enthalpy exchanger with infinite transfer coefficients: lightly-shaded, inlet state of stream 1, warm and humid; un-shaded, intersection point at low humidity ratio, hot and dry; dark-shaded, inlet state of stream 2, hot and humid.

$$\frac{\lambda_2}{\Gamma_j}\frac{\partial F_2}{\partial z}+\frac{\partial F_2}{\partial(\Phi/\beta_j)}=0. \tag{10}$$

The dimensionless wave speeds λ_i can be computed as the solution of the following quadratic expression [11]:

$$a_2a_5\lambda^2+(a_1a_4-a_2a_3-a_3)\lambda+a_3=0. \tag{11}$$

The a_i coefficients are derivatives of the thermodynamic state property relations

$$a_1=\left(\frac{\partial W_\mathrm{m}}{\partial t_\mathrm{m}}\right)_{w_\mathrm{f}} \quad a_2=\left(\frac{\partial W_\mathrm{m}}{\partial w_\mathrm{m}}\right)_{t_\mathrm{f}} \quad a_3=\left(\frac{\partial i_\mathrm{f}}{\partial t_\mathrm{f}}\right)_{w_\mathrm{f}}$$

$$a_4=\left(\frac{\partial i_\mathrm{f}}{\partial w_\mathrm{f}}\right)_{t_\mathrm{f}}-\left(\frac{\partial I_\mathrm{m}}{\partial W_\mathrm{m}}\right)_{t_\mathrm{f}} \quad a_5=\left(\frac{\partial I_\mathrm{m}}{\partial t_\mathrm{m}}\right)_{w_\mathrm{m}}.$$

The combined heat and mass transfer potentials were first introduced by Banks [13] and are frequently referred to as F-potentials. It is not necessary to compute absolute values of these potentials. The characteristic lines are lines in the Φ/β_j–z plane along which the F-potentials are constant and are defined by the following differential equation:

$$\frac{\mathrm{d}z}{\mathrm{d}(\Phi/\beta_j)}=\frac{\lambda_i}{\Gamma_j} \quad i=1,2. \tag{12}$$

Van den Bulck et al. [14] showed that, along the characteristic line, the following relation between temperature and humidity holds:

$$a_1a_3\,\mathrm{d}t_\mathrm{f}+(a_3-a_5\lambda_i)a_2\,\mathrm{d}w_\mathrm{f}=0. \tag{13}$$

Integration of equation (13) yields lines of constant F_i potential in a psychrometric plane as shown in Fig. 2 for a pair of air inlet states. Lines of constant F_1 resemble lines of constant enthalpy whereas lines of constant F_2 are similar to lines of constant relative humidity. The intersections of the lines of constant F-potential through the inlet states are called intersection points. The target of dehumidifier applications is dehumidification of stream 1 with regeneration of the matrix provided by stream 2, i.e. the desired outlet of stream 1 is the intersection point at low humidity ratio.

For enthalpy exchangers, high enthalpy effectiveness can be achieved if both potentials change. For the desiccants considered in this study (silica gel and molecular sieves) the wave speed λ_1 of the F_1-potential is about 10–100 times greater than λ_2. The transfer process can be illustrated in a wave diagram shown in Fig. 3. The slopes of the two wave fronts are determined by equation (12). Assuming that λ_1 and λ_2 are approximately constant for the process, the slopes are represented by the dimensionless parameter Γ which is the ratio of the 'matrix flow rate' to air flow rate. If the mass of the matrix and the air flow rate are fixed, Γ is a function only of rotation speed. Therefore, the rotation speed specifies the ratios of the dark-shaded to the un-shaded and lightly-shaded areas. Each area represents a distinct transfer zone with defined state properties as indicated in Fig. 3.

In Fig. 3, the value for Γ is high enough such that neither wave reaches the outlet of the regenerator. This reflection of both the first and second wave results in enthalpy exchange between the two flow streams. With infinite transfer coefficients and equal (balanced) air flows, the outlet state of stream 1 will be the same as the inlet of stream 2. The rotation speed of an enthalpy exchanger must be high enough to avoid the breakthrough of the first wave. Beyond that limit, a further increase of the rotation speed does not improve the performance of the device.

The equilibrium analysis described above is exact for ideal regenerators with equilibrium between the fluid streams and the matrix and holds approximately for regenerators with high transfer coefficients. Non-equilibrium systems with finite transfer coefficients tend to smear the transfer zones.

Equations (1)–(4) can also be uncoupled for the case of infinite rotation speed. Infinite rotation speed is equivalent to an infinitely high value of the dimensionless parameter Γ. After rearranging equations (1) and (3), the conservation laws for infinite rotation speed reduce such that the matrix water content and temperature become independent of the rotation angle (or time) and the partial differential equations simplify to a system of ordinary differential equations. These equations are still coupled since the enthalpy of the transferred water vapor is a function of both the temperature and humidity ratio. However, the sensitivity of the outlet air temperature to the number of transfer units for mass transfer is about two orders of magnitude less than the sensitivity to the number of transfer units for heat transfer [15]. This observation allows the latent term of the air enthalpy to be neglected which results in the following uncoupled equations:

$$\frac{dt_{f1}}{dz}-\frac{\dot{m}_{f2}}{\dot{m}_{f1}}\frac{dt_{f2}}{dz}=0 \quad (14)$$

$$\frac{dt_{f1}}{dz}+\frac{\dot{m}_{f2}}{\dot{m}_{f1}}\frac{dt_{f2}}{dz}-NTU_{t1}(t_{f2}-t_{f1})=0 \quad (15)$$

$$\frac{dw_{f1}}{dz}-\frac{\dot{m}_{f2}}{\dot{m}_{f1}}\frac{dw_{f2}}{dz}=0 \quad (16)$$

$$\frac{dw_{f1}}{dz}+\frac{\dot{m}_{f2}}{m_{f1}}\frac{dw_{f2}}{dz}-NTU_{w1}(w_{f2}-w_{f1})=0. \quad (17)$$

Equations (14) and (15) and (16) and (17) are each a set of independent equations and each is analogous to the conservation and transfer rate expressions of a counterflow direct type heat exchanger. The outlet states of the air streams can be computed conveniently using the humidity and temperature effectiveness

$$\varepsilon_w=\frac{w_{fj,\text{out}}-w_{fj,\text{in}}}{w_{f3-j,\text{out}}-w_{fj,\text{in}}} \quad (18)$$

$$\varepsilon_t=\frac{t_{fj,\text{out}}-t_{fj,\text{in}}}{t_{f3-j,\text{in}}-t_{fj,\text{in}}}. \quad (19)$$

These effectiveness factors can be computed using the relations for counterflow direct transfer heat exchangers along with the appropriate number of transfer units [1]. For the case of balanced flow streams, the correlations simplify to

$$\varepsilon_w=\frac{NTU}{1+NTU} \quad \text{where } NTU=\frac{NTU_w}{2} \quad (20)$$

$$\varepsilon_t=\frac{NTU}{1+NTU} \quad \text{where } NTU=Le\frac{NTU_w}{2}. \quad (21)$$

In the case of infinite rotation speed the slopes of both the first and second wave fronts become zero as shown in Fig. 4. Neither of the wave fronts moves completely through the matrix. The axial flow position of the wave fronts depends on the initial conditions. For balanced flow, the outlet of stream 1 is equal to the inlet state of stream 2 which is equivalent to a humidity and temperature effectiveness in equations (18) and (19) of unity. Comparison with Fig. 3 indicates that the outlet state of regenerators with infinite transfer coefficients reaches the inlet of the other stream even for finite rotation speed, as long as the second wave does not break through.

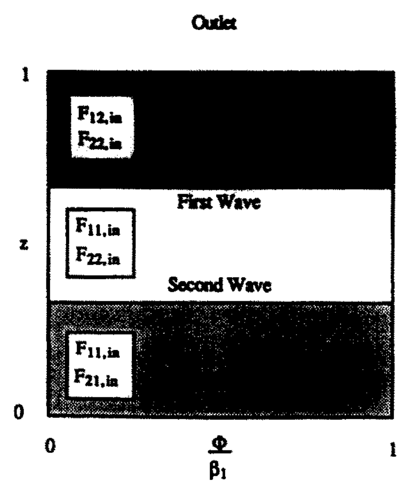

FIG. 4. Wave diagram for period 1 of an enthalpy exchanger with infinite rotation speed.

4. MODEL COMPARISON AND PARAMETRIC STUDIES

The range where the infinite rotation speed relations, equations (18)–(21), can be used to predict the outlet of an enthalpy exchanger with finite rotation speed and finite transfer coefficients is investigated using MOSHMX as a reference solution. Only balanced flows are considered. Further, the two period fractions, β_1 and β_2, are equal such that both the areas for heat and mass transfer and the transfer coefficients are the same for both periods. With these assumptions the performance of a regenerative heat and mass exchanger is determined by the values of Γ, NTU_t, Le, c_m, and the adsorption isotherm correlation.

Figure 5 shows the outlet states of stream 1 in a

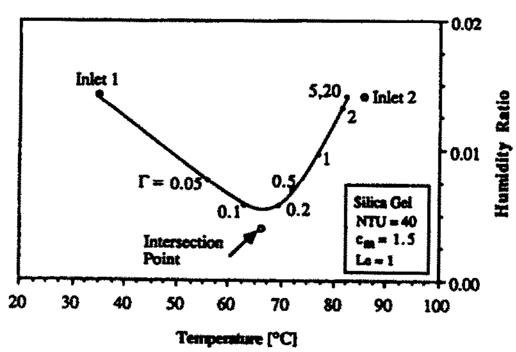

FIG. 5. Outlet states of stream 1 as computed by MOSHMX for high *NTU*.

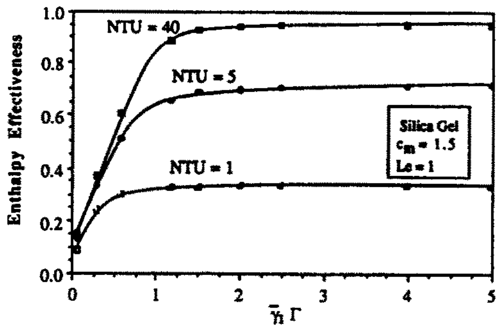

FIG. 6. Enthalpy effectiveness as computed by MOSHMX as a function of $\bar{\gamma}_1\Gamma$ and number of transfer units for the same inlet states as in Fig. 5.

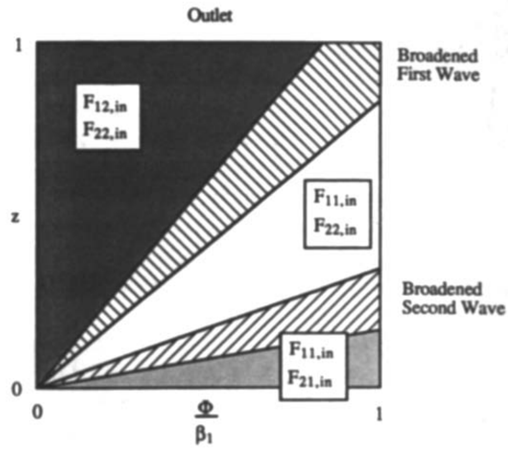

FIG. 7. Wave diagram with broadened wave fronts.

psychrometric chart as predicted by MOSHMX for a regenerative heat and mass exchanger with a high number of transfer units and various rotation speeds. The highest degree of dehumidification can be achieved at medium rotation speeds, equivalent to values of Γ between 0.1 and 0.2. In this range, the regenerator is operating as a dehumidifier with a breakthrough of the first wave and a reflection of the second wave. At higher values of Γ, the outlet of stream 1 approaches the inlet of stream 2 and the device is operating as an enthalpy exchanger. Because of the high value of *NTU*, the outlet temperature and humidity of the process stream are very close to that of the regeneration stream. The predicted outlet states do not change for values of Γ greater than 20.

A convenient measure for the approach of the outlet of stream 1 to the inlet of stream 2 is the enthalpy effectiveness which can be expressed as

$$\varepsilon_i = \frac{i_{f1,out} - i_{f1,in}}{i_{f2,in} - i_{f1,int}}.$$

Figure 6 shows the enthalpy effectiveness computed by MOSHMX as a function of the rotation speed and the number of transfer units. The product $\bar{\gamma}_1\Gamma$ was chosen as the independent axis. The average combined capacity ratio, $\bar{\gamma}_1$, was computed as the reciprocal of the non-dimensional wave speed λ_1 of the first wave at the average state of the two inlet streams, as suggested by Maclaine-cross [3]. Therefore, the independent axis can be interpreted as the reciprocal of the slope of the first wave front in a wave diagram illustrated in Fig. 3.

For the ideal case of sharp wave fronts in equilibrium exchange systems, the first wave is reflected if the slope of the first wave front is smaller than unity. Therefore, the enthalpy effectiveness becomes independent of the rotation speed if $\bar{\gamma}_1\Gamma > 1.0$ in the ideal case. For finite transfer coefficients, Fig. 6 indicates that for $\bar{\gamma}_1\Gamma > 1.5$ the enthalpy effectiveness can be considered to be a function of the number of transfer units only.

The wave diagrams shown in Fig. 3 are based on the assumption of constant λ_1 and λ_2 at each period. However, the values of the dimensionless wave speeds depend on the thermodynamic state properties of the air–water–desiccant system and must be computed by solving the quadratic expression in equation (11). The wave speeds vary as the matrix state changes. The two wave fronts propagating through the system broaden, as illustrated in Fig. 7.

The predicted outlet temperatures and humidities computed with the counterflow heat exchanger effectiveness correlations were compared to the finite difference solution. The errors are very large for low and medium rotation speeds but decrease rapidly as the reciprocal of the slope of the first wave front approaches the range where the regenerator is operating as an enthalpy exchanger. Figure 8 shows the error between the two methods. Therefore, the counterflow effectiveness correlations can be used to predict the outlet states of enthalpy exchangers if they are designed and operated at rotational speeds such that $\bar{\gamma}_1\Gamma > 1.5$.

The Lewis number is defined as the ratio of the number of transfer units for heat transfer to the number of transfer units for mass transfer and is a measure of the resistance to water vapor diffusion into the porous solid desiccant matrix. The value of the Lewis number for the solid desiccants is difficult to determine. Schultz [16] found the Lewis number of a silica gel dehumidifier to be near unity, while Van den Bulck found Lewis numbers of 3–4 for a similar system [12]. For *Le* of unity, the predicted outlet states lie on a straight line connecting the two inlet states. Figure 9 shows the predicted outlet states of an enthalpy exchanger for varying Lewis number. For increasing mass transfer resistance into the desiccant matrix, the outlet states are no longer on the line connecting the inlet states. For very large Lewis numbers the enthalpy exchanger operates as a rotary sensible heat exchanger where no mass is transferred between the flow streams and the humidity ratios at the outlets are equal to those at the inlets.

The effects of matrix thermal capacity and desiccant

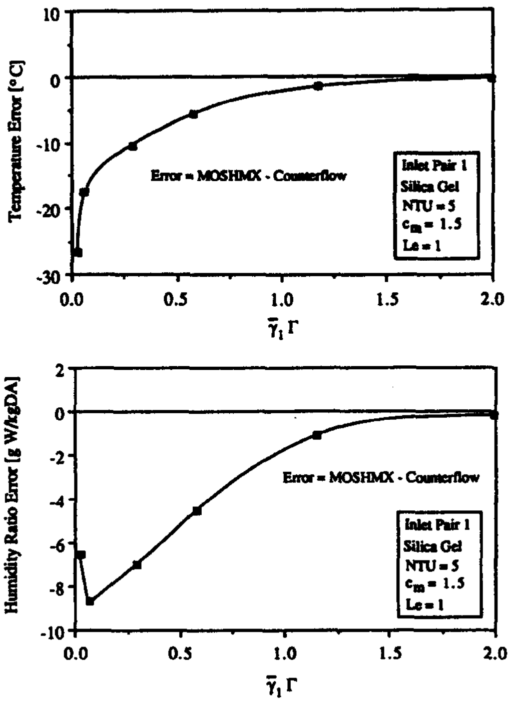

FIG. 8. Absolute errors in outlet states computed with counterflow heat exchanger correlations compared to MOSHMX.

isotherm have also been investigated [15]. The speed of the first wave is significantly affected by the thermal capacitance. However, for enthalpy exchange ($\bar{\gamma}_1\Gamma > 1.5$), the enthalpy effectiveness does not depend on the matrix thermal capacitance. The effect of desiccant type can also be correlated. Figure 10 shows the enthalpy effectiveness as a function of $\bar{\gamma}_1\Gamma > 1.5$ for silica gel and molecular sieves. The wave speeds for molecular sieves are slower than for silica gel and thus enthalpy exchangers with silica gel must rotate faster to achieve a value of $\bar{\gamma}_1\Gamma = 1.5$.

5. CONDENSATION AND FREEZING

A rotary regenerator with non-hygroscopic matrix usually transfers only sensible heat from the hot to the

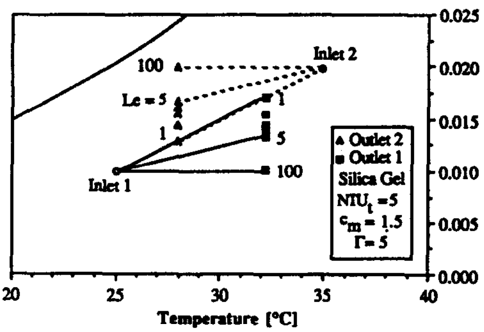

FIG. 9. Outlet states of streams 1 and 2 as computed using counterflow heat exchanger correlations for different *Le*.

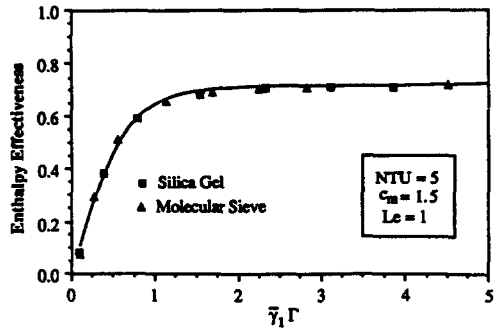

FIG. 10. Enthalpy effectiveness as computed by MOSHMX as a function of $\bar{\gamma}_1\Gamma$ for silica gel and a molecular sieve.

cold stream. However, if the dew point temperature of the hot and humid air stream is higher than the dry bulb temperature of the cold and dry stream, water vapor may condense on the matrix during one period and evaporate into the other air stream. Condensation may be advantageous in energy recovery applications of rotary sensible heat exchangers. However, the formation of ice on the matrix may result in poor performance if the flow channels of the regenerator become blocked.

Regenerators with non-hygroscopic matrices and water vapor transfer in the condensed phase have been investigated by Van Leersum [8, 10]. He used two different sets of differential equations and developed a finite difference solution scheme implemented in a program called REGENCOND. This program takes into account that, during steady state operation of the regenerator, parts of the matrix are covered with liquid bulk phase water whereas other parts are still dry.

Van Leersum compared the performance of REGENCOND with experimental results and obtained agreement within the experimental limitations [8]. If one regenerator inlet stream is very wet (46°C, 0.051 kgW/kgDA), the method did not converge to a steady state solution and led to the conclusion that the regenerator operates in an unsteady manner, consistent with his experimental results. The unsteady operation is due to the fact that, at one period, more water vapor condenses on the matrix than can be evaporated at the other period. This behavior violates the ***deposit condition*** postulated by Hausen [17] which requires that the water vapor deposited during one period must be evaporated before the end of the other period.

Holmberg [18] evaluated the condensation and frosting effects in non-hygroscopic and hygroscopic rotary heat exchangers. The limits for condensation and frosting and the resulting heat and mass exchange effectivenesses were determined for a specific exchanger. Typical operating conditions were also shown on psychrometric charts. Experimental results on frosting in enthalpy exchangers have been reported by Kruse and Vauth [19].

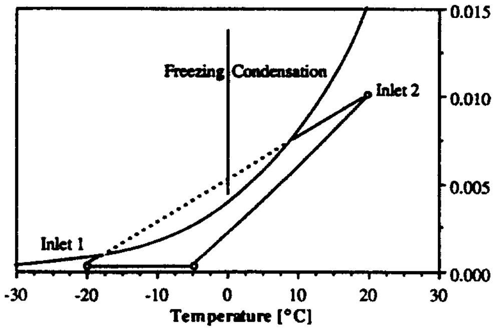

FIG. 11. Inlet conditions with freezing on a hygroscopic matrix.

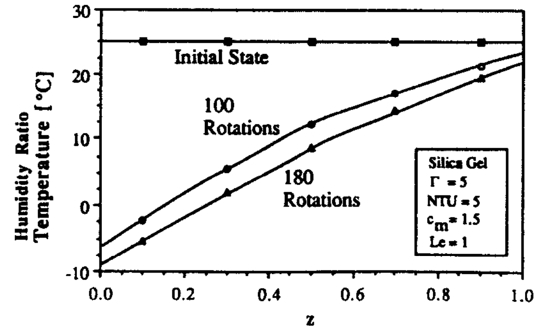

FIG. 12. Transient matrix temperature profiles of an enthalpy exchanger with one cold inlet stream at the beginning of the first period.

As shown in Fig. 9, the outlet states of an enthalpy exchanger with a hygroscopic matrix and unity Lewis number lie on a straight line connecting the inlet states, due to the fact that the temperature and water vapor effectiveness are equal. Therefore, condensation and freezing usually do not occur since the water vapor is transferred in an adsorbed rather than condensed phase. Only in cases where the line between the two inlet states intersects the saturation line must these phenomena be taken into consideration. An example for the case where water vapor transfer might occur in other than the adsorbed phase is shown in Fig. 11. The inlet conditions are for an energy recovery application of an enthalpy exchanger. The warm stream represents indoor conditions which are within the thermal comfort range specified by ASHRAE [11], whereas the cold stream is typical for outdoor conditions in very cold winter climates, such as Chicago, Illinois.

For the inlet conditions shown in Fig. 11, the finite difference method implemented in the version of MOSHMX available to the author would not converge since the amount of water vapor in the warm air stream and adsorbed by the desiccant cannot be removed from the enthalpy exchanger by the cold and dry air stream. Therefore, the matrix water content increases steadily until the maximum value is reached and water vapor is transferred from one period to the other in the condensed phase. If the matrix temperature is below 0°C, the water freezes on the matrix and blocks the flow channels of the regenerator.

MOSHMX was modified using the ideas of REGENCOND to model water vapor transfer in both the adsorbed and condensed phase. The conservation and transfer rate equations in equations (1)–(4) remain unchanged whereas the thermodynamic relationships depend on whether the hygroscopic matrix is saturated or not [15]:

matrix below saturation ($W_m \leqslant W_{max}$)

$$w_m = w_m(t_m, W_m) \tag{22a}$$

$$I_m = c_m t_m + c_{Lw} t_m W_m + \int_0^{W_m} (i_v - i_s)\, dW; \tag{23a}$$

matrix above saturation ($W_m > W_{max}$)

$$w_m = w_{sat}(t_m) \tag{22b}$$

$$I_m = c_m t_m + c_{Lw} t_m W_m + \int_0^{W_{max}} (i_v - i_s)\, dW; \tag{23b}$$

Equation (22a) reflects information contained in the adsorption isotherm whereas equation (22b) describes the fact that the solid desiccant is covered with condensed water (ice or liquid, depending on the temperature) if the maximum water content of the adsorbent is reached. The air state in equilibrium with the condensed phases can be described using the water vapor pressure curve as suggested by ASHRAE [11].

The algorithm implemented in MOSHMX uses successive substitution of the matrix states from one numerical cycle to another until the reversal condition is satisfied. This method essentially follows the changing matrix states from the initial values to the steady state solution. The modified version of MOSHMX was used in order to account for mass transfer in both the adsorbed and condensed phase. If the matrix water content becomes larger than its value for the maximum water uptake, condensed water or ice covers part of the desiccant. The susceptibility to ice formation is high at the cold side of the regenerator. Here the dry and cold air cools a matrix which has dehumidified the warm and humid exhaust air. Simulations are terminated if ice is on the matrix at the cold inlet for all angular positions which is equivalent to having all flow channels partially blocked.

Figures 12 and 13 illustrate the transient temperature and water content profiles of a silica gel enthalpy exchanger matrix just after the matrix switches from the warm to the cold stream using a pair of inlet states for which a straight line connecting the two inlet states in a psychrometric chart intersects the saturation line. Initially, the matrix is in equilibrium with the warm and humid stream (25°C, 0.010 kgW/kgDA). The cold air stream cools the matrix

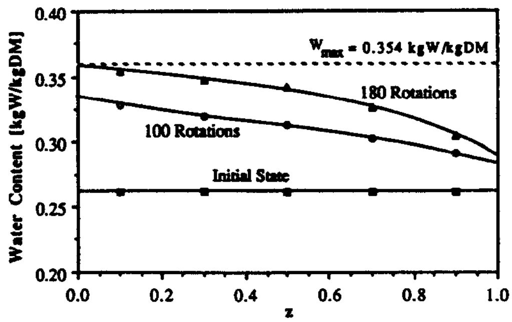

FIG. 13. Transient matrix water content profiles of an enthalpy exchanger with one cold inlet stream at the beginning of the first period.

below the freezing point of water. After about 160 rotations, ice starts building up at the inlet of the cold stream when the matrix is rotating from the warm to the cold period. For other angular positions, the matrix water content is still below its maximum value and ice formation does not occur. Finally, after 180 rotations, the matrix water content reaches the value for the maximum water uptake, which depends on the characteristics of the adsorbent. The entire matrix is covered with ice at the inlet of the cold stream and the simulation was terminated. The time required for freezing to occur was found to be dependent upon the adsorption isotherm of the desiccant. For example, the same calculations using molecular sieves resulted in ice buildup after six rotations.

If the inlet temperature of the cold stream is increased from -20 to $-10°C$, the finite difference algorithm converges to a steady state solution due to the fact that the line connecting the two inlet states does not intersect the saturation line on a psychrometric chart, as shown in Fig. 11. Therefore, after preheating the cold outdoor air stream, an enthalpy exchanger can be operated without ice formation on the matrix whereas a non-hygroscopic matrix would experience water vapor condensation or freezing under these conditions.

6. CONCLUSIONS

The objective of this study was to develop a computationally simple model of a solid desiccant air-to-air enthalpy exchanger. The theory of equilibrium exchange systems is used to show the operating conditions where enthalpy exchange between the two flow streams can be accomplished. The hyperbolic differential equations can be transformed to a set of uncoupled kinematic wave equations. To achieve optimum enthalpy exchange between the two air streams, the regenerator must be operated at conditions such that neither of the two transfer waves reaches the outlet of the enthalpy exchanger. Comparison with the numerical solution of the coupled equations for finite transfer coefficients shows that the product of two non-dimensional parameters, $\bar{\gamma}_1\Gamma$, must be greater than 1.5 in order to operate the enthalpy exchanger at a point where the enthalpy exchange effectiveness is determined only by the number of transfer units. $\bar{\gamma}_1\Gamma$ is most conveniently increased by increasing the rotation speed.

The case of infinite rotation speed was used as a basis for developing a computationally simple measure of performance. The outlet states can be computed using the ε–NTU correlations for counterflow direct transfer heat exchangers. It was found that these correlations are accurate at operating conditions where enthalpy exchange occurs; that is for $\bar{\gamma}_1\Gamma > 1.5$. The predicted outlet states lie, in the case of unity Lewis number, on a straight line connecting the outlet states.

A wide range of system parameters was investigated [15]. If the line connecting the two inlet states does not intersect the saturation line in a psychrometric chart the matrix parameters are not very important since reduced adsorptive capabilities of the matrix can be eliminated by rotating the enthalpy exchanger faster. The degree of performance in terms of enthalpy effectiveness is determined by the number of transfer units for heat transfer and the Lewis number.

The finite difference solution was modified in order to investigate the behavior of enthalpy exchangers at very cold outdoor conditions. Since enthalpy exchangers transfer water vapor in the adsorbed phase they are less susceptible to freezing than sensible heat exchangers in energy recovery applications in cold winter climates. Only for extreme conditions does the line between the inlet states intersect the saturation line and freezing is encountered. Under these conditions, freezeup can be avoided by slightly preheating the outdoor air.

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ANALYSE DES ECHANGEURS D'ENTHALPIE REGENERATEURS

Résumé—On développe un modèle numérique simple d'un échangeur d'enthalpie solide dessiccant–air. La théorie des systèmes d'échange en équilibre est utilisée pour établir les conditions opératoires sans lesquelles l'échange d'enthalpie entre les deux écoulements est réalisé. Pour atteindre le maximum d'échange d'enthalpie entre les deux courants d'air, le régénérateur doit opérer dans des conditions telles qu'aucune des deux ondes de transfert n'atteint la sortie de l'échangeur. Une comparaison avec la solution numérique des équations couplées pour les coefficients de transfert montre que le produit des deux paramètres adimensionnels $\bar{\gamma}_1\Gamma$, doit être supérieur à 1,5 de façon que l'échangeur d'enthalpie fonctionne sur un point où l'efficacité de l'échange enthalpique est déterminé seulement par le nombre d'unités de transfert. Le cas de la vitesse de rotation infinie est considéré comme une base pour permettre une mesure simple de la performance. Les états à la sortie sont calculés à partir de relations ε–*NTU* pour les échangeurs à transfert direct à contre-courant. On trouve que ces relations sont précises aux conditions opératoires lorsque se produit l'échange d'enthalpie. Les états calculés à la sortie se placent, dans le cas du nombre de Lewis égal à un, sur une ligne droite reliant les états de sortie.

UNTERSUCHUNG VON REGENERATIVEN ENTHALPIE-AUSTAUSCHERN

Zusammenfassung—Es wird ein rechnerisch einfaches Modell eines massiven Luft–Luft-Enthalpie-Austauschers entwickelt. Die Theorie von Austausch-Systemen im Gleichgewicht wird benutzt, um die Betriebsbedingungen, bei denen Enthalpie-Austausch zwischen den beiden Fluidströmen bewerkstelligt werden kann, zu begründen. Um den maximalen Enthalpie-Austausch zwischen den beiden Luftströmen zu erreichen, sollte der Regenerator unter solchen Bedingungen betrieben werden, daß keine der beiden Übertragungs-Wellen den Auslaß des Enthalpie-Austauschers erreicht. Ein Vergleich mit der numerischen Lösung der gekoppelten Gleichungen für endliche Übertragungs-Koeffizienten zeigt, daß das Produkt von zwei dimensionslosen Größen, $\bar{\gamma}_1\Gamma$, größer als 1,5 sein sollte, damit der Enthalpie-Austauscher in einem Punkt betrieben wird, an dem der Enthalpie-Austauscher-Wirkungsgrad lediglich von der Anzahl der Übertragungseinheiten (*NTU*) beeinflußt wird. Der Fall einer unendlich großen Drehzahl wird als Bezug für einen einfach zu berechnenden Wirkungsgrad verwendet. Die Auslaß-Zustände werden mit Hilfe der ε–*NTU*-Korrelationen für Gegenstrom-Wärmeaustauscher mit direkter Übertragung berechnet. Es zeigte sich, daß diese Korrelationen bei denjenigen Betriebsbedingungen genau sind, bei denen der Enthalpie-Austausch auftritt. Die vorausgesagten Auslaß-Zustände liegen für die Lewis-Zahl Eins auf einer geraden Linie, welche die Auslaß-Zustände verbindet.

АНАЛИЗ РЕГЕНЕРАТИВНЫХ ТЕПЛООБМЕННИКОВ

Аннотация—Разработана простая расчетная модель твердотельного осушающего воздухо–воздушного теплообменника. С целью определения рабочего режима, при котором может осуществляться обмен энтальпией между двумя потоками, используется теория систем равновесного обмена. Для достижения максимального обмена энтальпией между двумя воздушными потоками регенератор должен работать при таких условиях, когда ни одна из двух волн переноса не достигает выхода из теплообменника. Сравнение с численным решением связанных уравнений для ограниченных коэффициентов переноса показывает, что для того, чтобы эффективность обмена энтальпией определялась только количеством переносимых единих. произведение двух безразмерных параметров $\bar{\gamma}_1\Gamma$ должно превышать 1,5. В качестве основы для разработки простого расчетного критерия рабочих характеристик рассматривался случай неограниченной скорости вращения. Состояния на выходе рассчитаны с использованием соотношений ε–*NTU* (количество переносимых единиц) для противоточных теплообменников с прямым переносом. Найдено, что данные соотношения являются точными при рабочих условиях с обменом энтальпией. В случае, когда число Льюиса равно единице, рассчитанные значения состояний на выходе находятся на соединяющей их прямой.